ST553 HW7

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Problem 1

Here we are given the following random effects model with a balanced design:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

where $\alpha_i \sim N(0, \sigma_\alpha^2)$ and $\epsilon_{ij} \sim N(0, \sigma^2)$ and $\alpha_i \perp \epsilon_{ij}$

Our goal is to derive the expected value of MSA.

MSA is given as $\frac{SSA}{a-1}$ where a is the number of levels of the random effect. We are also given that SSA can be represented in a quadratic form as $SSA = Y^T(H - \frac{1}{N}J)Y$ where H is the projection matrix and J is an N x N matrix of 1s.

Problem 2

We have data for *one* random factor in this model, the randomly selected bull.

Our model will look like

$$y_{ik} = \mu + \alpha_i + \epsilon_{ik}$$

where

$$\alpha_i \sim N(0, \sigma^2_\alpha)$$
$$\epsilon_{ik} \sim N(0, \sigma^2)$$
$$\text{all effects independent from one another}$$

and y_{ik} is the growth of the kth calf from the ith bull, μ is the overall mean, α_i corresponds to the random effect of the ith bull, and ϵ_{ik} is the random error of the kth calf's gains.

Note that in the problem as given in the textbook, we should have two random effects: the bull and the cow. The parameterization for that model is given in the appendix of this assignment.

Type 3 Analysis of Variance								
Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
bull	3	0.139233	0.046411	Var(Residual) + 6 Var(bull)	MS(Residual)	20	2.91	0.0597
Residual	20	0.318900	0.015945	Var(Residual)	.	.	.	.

Our null hypothesis is that $H_0 : \sigma^2_{alpha} = 0$, or in less technical words, that there is no effect of bull on calf weight gain. Out alternative hypothesis is that there is some effect on calf weight gain.

Our test statistic is an F-statistic (2.91) with 3 and 20 degrees of freedom. Our p-value is 0.597, so we **fail to reject** the null hypothesis at the $\alpha = .05$ level, but the p-value is still somewhat low and should be not entirely uninteresting to a cattle farmer or scientist.

In a single sentence: we found moderately significant evidence that bull sire has any effect on daily calf weight gain.

Here are the variance component estimates using REML. *Interestingly, they are the same as the MOM estimates.*

Covariance Parameter Estimates	
Cov Parm	Estimate
bull	0.005078
Residual	0.01594

Problem 3

We are given the following SAS output and asked to find the F statistic and degrees of freedom in the denominator for a test of $H_0 : \sigma_\beta^2 = 0$:

Type 3 Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	Expected Mean Square
A	2	0.454877	0.227439	Var(Residual) + 16.977 Var(A*B) + 33.955 Var(A)
B	1	11.837141	11.837141	Var(Residual) + 16.916 Var(A*B) + 50.747 Var(B)
A*B	2	0.384226	0.192113	Var(Residual) + 16.977 Var(A*B)
Residual	97	274.720170	2.832167	Var(Residual)

Appendix

For problem 2 should have *two* random factors in this model, the randomly selected bull and the randomly selected cow.

Our model will look like

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where

$$\begin{aligned}
\alpha_i &\sim N(0, \sigma^2_\alpha) \\
\beta_j &\sim N(0, \sigma^2_\beta) \\
(\alpha\beta)_{ij} &\sim N(0, \sigma^2_{\alpha\beta}) \\
\epsilon_{ijk} &\sim N(0, \sigma^2) \\
\text{all effects independent from one another}
\end{aligned}$$

and y_{ijk} is the growth of the *kth* calf from the *ith* bull and *jth* cow, μ is the overall mean, α_i corresponds to the random effect of the *ith* bull, β_j corresponds to the effect of the *jth* cow, $(\alpha\beta)_{ij}$ is the random interaction effect of *ith* bull and *jth* cow, and ϵ_{ijk} is the random error of the *kth* calf's gains.